The Maximum Optical Depth Towards Bulge Stars From Axisymmetric Models of the Milky Way

Konrad Kuijken[1]

Kapteyn Instituut, PO Box 800, 9700 AV Groningen, The Netherlands

ABSTRACT

It has been known that recent microlensing results towards the bulge imply mass densities that are surprisingly high given dynamical constraints on the Milky Way mass distribution. We derive the maximum optical depth towards the bulge that may be generated by axisymmetric structures in the Milky Way, and show that observations are close to surpassing these limits. This result argues in favor of a bar as a source of significantly enhanced microlensing. Several of the bar models in the literature are discussed.

Subject headings: Galaxy: kinematics and dynamics — Galaxy: structure — gravitational lensing

[1]Visiting Scientist, Dept. of Theoretical Physics, University of the Basque Country

1. Introduction

Large surveys for microlensing events in the Galaxy have grown into a serious tool for evaluating models of the mass distribution of the Milky Way, as is evident from the recent results of the MACHO (Alcock et al. 1997), OGLE (Udalski et al 1994), EROS (Renault et al. 1996) and DUO (Alard, Mao & Guibert 1995) surveys. Microlensing occurs whenever the light from a distant star passes sufficiently close to an intervening object (the 'lens') that its path is significantly deflected. The deflection results in a focusing of the source star's rays, which may be detected as an apparent brightening of the source with a characteristic light curve. Typically the brightening lasts for a number of days or months before the source and lens proper motions spoil the alignment. Both the optical depth τ, which is the average fraction of the sources that is being amplified more than a factor of 1.34 at any given time, and the average duration of the individual events, depend on the distribution and masses of lensing objects between the observer and the sources (Paczyński 1986).

In this paper we concentrate on microlensing towards the bulge of the Milky Way. Both the MACHO and OGLE teams find that the optical depth in this direction is rather higher than had been expected initially: the optical depths reported are in the range of 3–6$\times 10^{-6}$, roughly twice the expected value based on standard Galaxy models. Although initially this discrepancy was attributed to our Galaxy's central bar (Paczyński et al. 1994, Zhao, Spergel & Rich 1995, Zhao, Rich & Spergel 1996), evidence for which had been accumulating for some time (e.g. Blitz & Spergel 1991, Weinberg 1992; for a review, see Kuijken 1996), the most recent models for the bar in fact do little to enhance the expected optical depth over that expected from axisymmetric models (Bissantz et al. 1996, Nikolaev & Weinberg 1997). It therefore seems opportune to return to the axisymmetric mass models once more.

In this letter we derive a strong limit on axisymmetric models: a given optical depth towards the bulge comes with a minimum contribution to the Galactic circular speed at the sun. It turns out that the current values for the optical depth towards the bulge are already close to implying a circular speed higher than that observed.

The only result from microlensing theory that is relevant for this discussion is the optical depth τ to a source at distance L, due to a distribution of lenses with mass density $\rho(D)$ at distance D down the line of sight:

$$\tau = \frac{4\pi G}{c^2}\int_0^L \rho(D)\frac{D(L-D)}{L}dD. \tag{1}$$

τ is independent of the mass function $f(m)$ of the lenses, though the rate and detectability of microlensing events are not: the survey teams correct for detection (in)efficiency in their analyses.

2. Can Axisymmetric Mass Components Explain Bulge Microlensing?

Axisymmetric density distributions can be viewed as constructed from concentric rings. We therefore start by considering the relation between the mass and the optical depth of a ring. We assume that the sources are on the Galactic minor axis, at a distance of $R_0 = 8$kpc, and not mixed spatially with the ring; these results are thus appropriate for the microlensing of the bulge clump giants (Alcock et al. 1997).

2.1. Optical Depth from a Ring

Consider a ring of uniform surface density with radius $R < R_0$ and radial width $\Delta R \ll R$. Its optical thickness towards bulge sources near the Galactic minor axis, at latitude b radians (assumed small), is (cf. eq. 1)

$$\tau_{\rm ring} = \frac{4\pi G}{c^2}\frac{(R_0-R)R}{R_0}\rho(z)\Delta R. \tag{2}$$

Here ρ is the density where the line of sight intersects the ring, at height $z = (R_0 - R)b$ from the plane. Now, if the ring is axisymmetric with uniform surface density Σ, then its mass is

$$M_{\rm ring} = 2\pi R\Delta R\Sigma = \frac{c^2\tau_{\rm ring}R_0 b}{2G}\left(\frac{\Sigma}{z\rho(z)}\right) \tag{3}$$

(using $R_0 - R = z/b$). The bracketed factor is dimensionless, and depends only on the shape of the vertical density profile of the ring, not on the total mass of the ring. We will call it the vertical shape factor F_z. Its denominator tends to zero at small and large z (provided the ring has finite surface mass density), and therefore this geometrical factor has a minimum at some intermediate z. Hence equation 3 can be used to derive a lower limit on the mass of an axisymmetric ring of a given optical depth τ.

The lowest vertical shape factor for monotonic $\rho(|z|)$ is obtained when the density is constant up to a height $a > z$, and zero beyond. In this case $\Sigma = 2a\rho$, and $F_z \geq 2$. More realistic density profiles yield values considerably larger, though. Table 1 lists the lower limits to F_z for some different forms of $\rho(z)$. In light of those results, we adopt $F_z \gtrsim 4.0$.

The lower mass limit of Eq. 3 can be recast as a circular velocity at the solar circle: approximating the circular speed due to a ring by its monopole term (itself a lower limit), we obtain

$$v_{\rm circ}^2 \geq \frac{GM_{\rm ring}}{R_0} = \frac{c^2\tau_{\rm ring}b}{2}F_z \gtrsim 2.0c^2\tau_{\rm ring}b. \qquad (4)$$

Since the radius of the ring is no longer explicit in this equation, this limit applies equally to superpositions of rings, i.e., axisymmetric mass distributions. Scaling to observed parameters, we obtain

$$v_{\rm circ}^2 \geq \left(210{\rm km\,s^{-1}}\right)^2 \left(\frac{\tau_{-6}}{4.0}\right)\left(\frac{b}{3.5^\circ}\right)\left(\frac{F_z^{(\rm min)}}{4.0}\right) \quad (5)$$

as the minimum circular speed generated at the solar position by axisymmetrically distributed matter responsible for an optical depth of $\tau_{-6} \times 10^{-6}$ towards bulge sources at Galactic latitude b. A radially extended mass distribution, in order to lens optimally, needs to have F_z close to its minimum at all radii: this is achieved when the scale height of the mass is proportional to $R_0 - R$, i.e., tapers linearly to zero at the solar radius.

2.2. Specific Disk Models

We know more about the Milky Way rotation curve than its value at the solar position: we also know that the circular speed does not greatly exceed the local value at any point in the inner disk of the Galaxy. We now investigate how this extra constraint affects the maximum disk optical depth, by considering various models for the disk mass distribution and raising their mass until their rotation curve reaches $200{\rm km\,s^{-1}}$ at any point. We consider four functional forms for the disk surface mass density, whose rotation curves are shown in Fig. 1. We will also compare these models to solar-neighborhood normalized models, where the surface mass density in stars is $\sim 35{\rm M_\odot pc^{-2}}$ (Kuijken & Gilmore 1989). Note that the total surface density of matter within 1 kpc of the plane, $71{\rm M_\odot pc^{-2}}$ (Kuijken & Gilmore 1989) is consistent with such maximal disk models only for short ($\lesssim$ 2.5kpc) scale length models (Sackett 1997).

The traditional disk model is a radially exponential disk, $\Sigma(R) \propto \exp(-R/h)$. We consider two vertical density profile shapes, a vertical exponential and a ${\rm sech}^2(z/a)$ 'isothermal' model. If the scale height is small, the rotation curve peaks at a radius of 2.2 scale lengths at the value $0.62(GM/h)^{1/2}$ (Freeman 1970). Figure 2 shows the optical depth from such disks towards the Galactic minor axis at a latitude of $b = 3.5^\circ$, for various scale heights and lengths.

For comparison, we consider also two variations on the exponential disk: a less centrally concentrated model with $\Sigma(R) \propto R\exp(-R/h)$ (e.g., Caldwell & Ostriker 1981), and a more concentrated one obtained by adding a second exponential disk of a quarter the scale length, and one eighth the mass of the main component. As may be seen from Figs. 3 and 4, the maximum optical depth from such disks is comparable to the exponential disk.

Finally, we consider a Mestel disk (Mestel 1963), which has surface mass density $\Sigma(R) = V^2/(2\pi GR)$, and whose circular speed is everywhere equal to V. Such a model microlenses more efficiently than an exponential disk since it can satisfy our rotation curve constraint at every radius simultaneously. The maximum optical depth from a Mestel disk towards $b = 3.5^\circ$ turns out to be 2.5×10^{-6} for a vertically ${\rm sech}^2(z)$ mass distribution.

We summarize the results for the various disk models in Table 2. We give two numbers for each model: the maximal optical depth for each model assuming constant scale height, and the (higher) optical depth that can be achieved by tapering the disk scale height down to zero at the solar position, in proportion to $(R_0 - R)$.

Table 1: The minimum vertical shape factor $F_z = \Sigma/(z\rho)$ for different vertical density profile shapes.

$\rho(z)$	$F_z^{(\rm min)}$	Attained at $z =$
constant to $z = a$	2	a
$\exp(-\lvert z\rvert/a)$	$2e$	a
${\rm sech}(z/a)$	4.7	$1.2a$
${\rm sech}^2(z/a)$	4.5	$0.8a$
$\exp(-z^2/a^2)$	4.1	$0.7a$

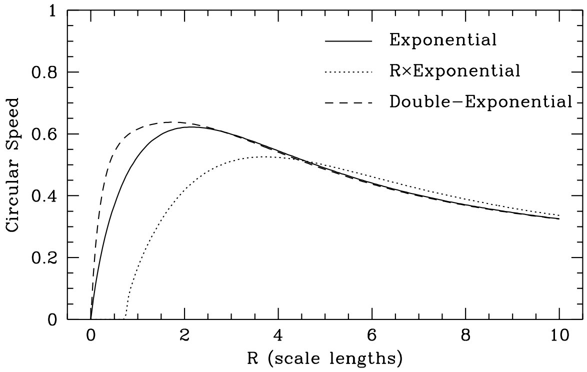

Fig. 1.— The rotation curves for three types of disk models considered, in units of $(GM/h)^{1/2}$.

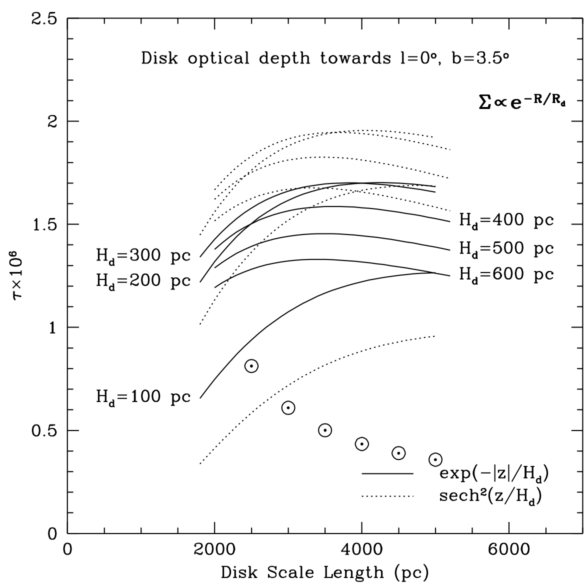

Fig. 2.— The optical depth towards sources at the Galactic Minor axis from radially exponential disks with maximum circular speed 200km s^{-1}, and differing scale heights and lengths. The solid lines refer to vertically exponential disks, the dotted ones to sech^2 disks. Sun symbols show the optical depths from disks normalized to the 'locally identified' stellar surface density of 35M$_\odot$pc^{-2} at the solar position, vertically exponential with scale height 300pc.

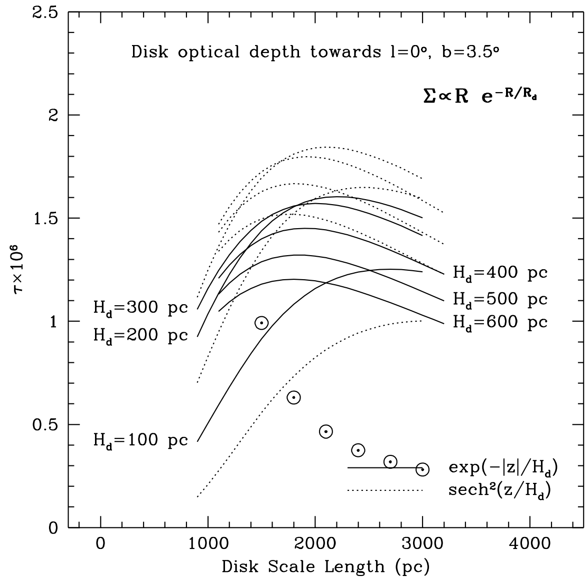

Fig. 3.— As Figure 2, but with a central hole in the disk mass profile.

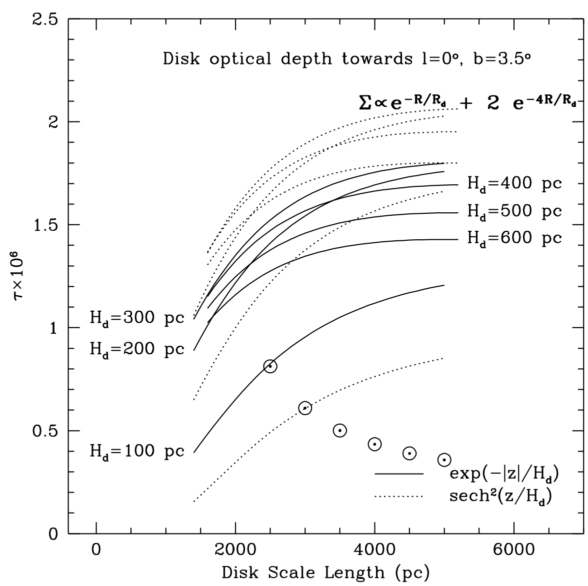

Fig. 4.— As Figure 2, but with a more concentrated mass profile.

3. Discussion

The calculations presented above show that axisymmetric models of the Galaxy cannot avoid high circular velocities at the sun if the optical depth towards the Galactic minor axis is as high as is being reported. The current numbers are in fact so high as to exceed the maximal disk values derived in Table 2. They are also in conflict with the claimed detection of the dark halo towards the LMC (Alcock et al. 1996, Renault et al. 1996), since the small disk scale heights (~ 300pc) required for bulge microlensing would produce virtually no microlensing towards the LMC. If indeed the optical depth is too high, then non-axisymmetric models, particularly the Galactic bar, should be considered.

Several in-depth studies of microlensing by the central bulge/bar have been carried out already. While the recent investigations by the Oxford group (Binney et al. 1996; Bissantz et al. 1996) and by Nikolaev & Weinberg 1997 conclude that the enhancement in optical depth over a standard bulge model (Kent 1992) is small, the earlier analysis by Zhao et al. (1995,1996) did find a significant enhancement in τ. These differences illustrate the difficulty in disentangling the structure of the inner Galaxy from our view point in the disk.

The Zhao et al. and Oxford studies are based on the COBE/DIRBE map of near-infrared emissions, the asymmetries of which are used to infer the bar's orientation and shape from perspective effects. Zhao et al. based their model on the fitting to the DIRBE data by Dwek et al. 1995, whose best-fit model is quite elongated, with major axis close to the line of sight. This model is therefore efficient at microlensing towards the bulge, since it places most of its mass density between us and the bulge. The Oxford group, on the other hand, use a Lucy deprojection of the emission to infer the bar properties. They find a much shorter bar, which consequently generates less microlensing. Nikolaev & Weinberg derive their bar parameters from star counts of IRAS AGB sources. Their bar is considerably longer than the COBE-derived ones, but it is quite broad, providing only a modest enhancement in lensing over axisymmetric models.

Table 2: Maximum optical depth towards stars on the Galactic minor axis at latitude $b = 3.5°$, for various disk surface density profiles $\Sigma(R)$. All models have been normalized to have a peak circular speed of $200 \mathrm{km\,s^{-1}}$, and have vertical density $\propto \mathrm{sech}^2(z)$. The optical depths for models whose scale height is radially constant, and tapered $\propto (R_0 - R)$ are shown.

Disk $\Sigma(R)$	$\tau_{\rm const} \times 10^6$	$\tau_{\rm tapered} \times 10^6$
$\propto \exp(-R/a)$	2.0	2.6
$\propto R\exp(-R/a)$	1.9	2.4
$\propto \exp(-R/a) + 2\exp(-R/4a)$	2.1	2.7
$\propto R^{-1}$	2.5	3.3

It is difficult to prefer one of these models over another. The Zhao et al. model is a simple parametric fit to the COBE data, whereas the Oxford model is completely non-parametric. In fact, the Oxford inversion is puzzlingly robust: different starting conditions result in very similar solutions, in spite of the fact that a three-dimensional emissivity model is reconstructed out of a two-dimensional projected surface brightness map! One assumption made is, however, crucial: the models are taken to be 8-fold symmetric, i.e., the bar is modeled as reflection symmetric in three orthogonal planes (it is this symmetry that allows the observed left-right differences in the map to be interpreted as perspective effects). In reality, even the most regular bars observed do not exhibit this symmetry: bars are generally somewhat twisted, and often connect onto spiral arms joining at the ends. As Binney et al. themselves recognize, even a 4-fold symmetry (reflection in Galactic plane and through the Galactic center) is insufficient to allow a stable inversion.

The Weinberg bar may be stronger than it appears in the analysis of IRAS sources, since a relatively small number of harmonics was used in modeling the star counts ($m = 0, 2, 4$ azimuthal harmonics). It is therefore possible that the true density enhancement towards the bulge is higher than modeled. This bar is also clearly larger than the COBE models, suggesting that the Galaxy may well be a bar-within-a-bar galaxy, a type of system which is relatively common. In that case, the COBE models may need to take account of this extra foreground component.

4. Conclusion

We have shown that if the mass responsible for the microlensing towards the Galactic bulge is distributed axisymmetrically, the high optical depths τ being reported may be translated into simple but significant lower limits on the circular speed at the solar position

due to lensing matter:

$$v_c^2 \geq 2.0c^2\tau b \tag{6}$$

where b is the galactic latitude (in radians) of the sources. This limit evaluates to $v_c \geq 210\mathrm{km\,s^{-1}}$ for an optical depth of 4×10^{-6} at $b = 3.5°$, very close to the actual rotation speed of $200{\pm}20\mathrm{km\,s^{-1}}$ (Merrifield 1992).

This limit is approached when the lensing matter has a scale height that is well matched to the height of the line of sight towards the stars being monitored, otherwise even higher circular speeds are implied. More restricted, but realistic, models for the mass distribution fail to generate optical depths above $2.5{\times}10^{-6}$ without requiring circular speeds in excess of $200\mathrm{km\,s^{-1}}$.

This result comes close to ruling out purely axisymmetric models for the mass distribution in the inner Galaxy. Those models left require a very flattened distribution of the Galactic mass, of thickness $\sim$ 300pc, contrary to the conclusions drawn from the microlensing experiments towards the Large Magellanic Cloud, which favor a round dark matter distribution. A significant enhancement in τ from a central bar appears to be a more plausible explanation.

It is a pleasure to thank Penny Sackett for discussions and a careful reading of the manuscript.

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